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THEORETICAL STUDY OF LAMINAR FILM CONDENSATION OF FLOWING VAPOUR

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Аннотация—Рассмотрена ламинарная пленочная конденсация движущегося пара на плоской пластине и на наружной поверхности цилиндра при поперечном обтекании. Показана определяющая роль переноса импульса конденсирующейся массой вещества в гидродинамике рассмотренных процессов. Полученные результаты сопоставлены с имеющимися экспериментальными данными.

NOMENCLATURE

x, distance along the plate;
y, distance normal to the plate;
n, distance normal to the cylinder;
φ, angle counted from the frontal point of the cylinder;
U, velocity in x-direction;
V_0, velocity of vapour entering into the film;
C_f, friction coefficient;
C_f^*, factor accounting for the inertia component of friction in a suction flow;
C_Q, flow coefficient;
τ_0, shearing stress at the friction surface;
j, mass flow of vapour to the film;
Re, $= (U_\infty X/\nu_V)$, Reynolds number;
Re_D, $= (U_\infty D/\nu_V)$, Reynolds number for a flow round a cylinder;
N, $= (\lambda\,\Delta t/r\mu)$;
Nu_D, $= (\bar{\alpha}D/\lambda)$;
η, $\dfrac{\rho\, U_\infty\, D}{\mu}\left[1 + \sqrt{\left(1 + 1{\cdot}69\,\dfrac{Dg}{U_\infty^2\, N}\right)}\right]$;
λ, thermal conductivity;
ρ, density;
γ, specific weight;
μ, dynamic viscosity;
C_p, heat capacity at constant pressure;
r, latent heat of condensation;
g, gravitational acceleration;
$\mathscr{L}$, plate length;
D, cylinder diameter;
t, temperature;
Δt, temperature drop between the cooling wall and saturated vapour;
q, heat flux;
δ, film thickness;
α, heat-transfer coefficient.

Subscripts

V, vapour flow;
∞, at infinity;
w, at the cooling surface;
s, at the interface;
x, at X-distance from the leading edge of the plate;
φ, at the distance corresponding to φ-angle from the frontal cylinder point.

Note: Symbols which denote values referring to the liquid phase are used without subscripts.

1. INTRODUCTION

FILM condensation of flowing vapour has been the subject of a number of theoretical and experimental works.

The problem was originally treated by Nusselt [1] who analysed laminar film condensation in a vapour flow over a vertical surface. Nusselt's theory was further developed in references [2–4]. Laminar film condensation in a transverse vapour flow on the external surface of a horizontal cylinder is discussed in references [5–7].

The knowledge of the mechanism of interaction at the phase interface is clearly of fundamental importance for the analysis of the problem considered. The authors of references [1–7] use the assumption that interfacial shear in the condensation of a vapour flow is the same as that at a dry impermeable surface in a flow of a non-condensing gas. This implies that, in the works just referred to, the effect of the momentum transfer caused by the mass of the condensing vapour in the direction normal to the flow is neglected. Such an approach does not agree with the actual hydrodynamic pattern of the process. As shown in the present work, in reality the phase conversion which is inseparably linked up with the mass flow across the interface is the dominant factor in the dynamics of the process.

In reference [8] an attempt was made to prove that the effect of the momentum transferred by the mass of the condensate in film condensation of flowing vapour is negligible. In that work the authors referred to the fact that the velocity field has no discontinuity at the interface. Further, this approach was recognized by some workers who discussed reference [8]. It should be noticed, however, that this concept contradicts some fundamentals of the boundary-layer theory.

On the basis of the two-phase boundary-layer equations, solutions are presented by Cess [9] and Koh [10] for film condensation in a laminar flow of saturated vapour over an isothermal plate in the absence of body forces. For the solution of these equations, in reference [9] a number of simplifying assumptions were made. According to Cess [9], these assumptions should be equivalent to neglecting the inertia forces and convection in a condensate film. However, as the results have shown, the implications of the assumptions of Cess [9] are not exhausted by neglecting the above effects. In reference [10] the results of the numerical solution are given for the system of the basic equations presented by Cess. These solutions are obtained without any simplifying assumptions. It should be noted that some main results of the work evidently contradict each other. For example, according to Koh [10], the condensate velocity in a film at an equal distance from the plate decreases streamwise along the surface according to the law

$$U_x \sim \frac{1}{\sqrt{x}}.$$

This implies that the inertia forces in a condensate film are orientated in the direction of the flow. It is evident that in view of all that has been said, the neglect of the inertia forces in the solution of the problem should lead to an underestimation of the heat-transfer rate. However, the comparison in Fig. 8(a) of reference [10] shows that the neglect of the inertia forces gives in fact overestimated values of the heat-transfer rate. Also, the main conclusion of Koh [10], that convection has a considerable effect on the heat-transfer rate, seems doubtful. In fact, as shown in the work mentioned, the effect of this factor is not very important.

Together with the theoretical studies, experimental studies on film condensation in vapour flow have also been carried out. With regard to the problem considered, the work of Jakob [11] is of great interest. The measured velocity and temperature fields presented in this work reveal, contrary to Nusselt's conceptions, the essential effect of phase conversion on the dynamics of the vapour flow. The measurements of the mean heat-transfer coefficients in Jakob *et al.* [2] have shown that the actual effect of temperature drop differs sharply from that predicted by Nusselt's theory.

References [12–14] treat condensation heat transfer in a water vapour flow over the external surface of a single horizontal tube and across a bundle of horizontal tubes. The results of these works have shown that the analytical solutions based on Nusselt's conceptions do not describe even qualitatively the character of the process.

According to these experimental data the vapour motion has a more essential effect on the heat-transfer rate than predicted by the analytical solutions.

In the following part, the theoretical results of some cases of laminar condensation in a vapour flow are reported. A brief account of the particular data has already been published [15].

2. CONDENSATION ON A FLAT PLATE

Vapour flow and friction at the interface

The conditions for the development of the boundary layer in a flow of condensing vapour differ from the conditions of a non-condensing gas flow over an impermeable plate. The main difference between these effects is caused by the mechanism of phase conversion itself. Since the condensation process is inseparably linked with the mass flow across the interface, the boundary-layer development is accompanied by a continuous mass intake through the friction surface. Evidently, all points of the surface in the flow take part in the intake of vapour which occurs in the form of condensation. This cannot be achieved with forced suction of the boundary layer by the well-known methods described in the literature. As it is known, the boundary layer near the leading edge of the plate is laminar, irrespective of the condition of the flow. Suction stabilizes the laminar boundary layer and under certain conditions it eliminates completely the possibility of transition to a turbulent flow in the boundary layer. According to Schlichting [16], the condition of complete stability of the laminar boundary layer flow is expressed by

$$C_Q = -\frac{V_0}{U_\infty} > 1{\cdot}18 \times 10^{-4}. \quad (1)$$

The rate of suction sufficient to prevent the transition of a laminar boundary layer to a turbulent one, and satisfying the inequality (1) is very small. The vapour "suction" in the actual cases of condensation considerably exceeds this value. For example, calculations show that in the case of water-vapour condensation with a flow velocity of 100 m/s and pressure of 0·35 N/cm^2 (pressure, usual for power condensers), heat flux of 650 W/m^2 gives rise to a rate of drawing off sufficient to prevent transition to a turbulent flow in the vapour phase. It is of interest to note that according to the present data, the temperature drop between saturated vapour and the cooling wall which is necessary for the creation of this heat flux is several hundredths of a degree.

Proceeding from the above, in the present analysis the flow in the vapour phase is assumed to be laminar over the whole surface in the flow.

The presence of a mass flow through the phase interface determines also the type of friction between the vapour flow and the condensate film.

The momentum equation for an incompressible boundary layer with suction after some transformations may be expressed as follows:

$$\tau_0 = C_f^* \frac{\rho_V U_\infty^2}{2} + 2C_Q \frac{\rho_V U_\infty^2}{2} \quad (2)$$

As is known, the first component of the total friction from (2) tends to decrease sharply as the distance from the edge of the plane increases. It is also known, that C_f^* is always smaller than the drag coefficient in a laminar flow over a plate without suction.

With sufficiently high suction rates, the shearing stress on the friction surface depends mainly on the momentum transferred by the suction mass and may be determined from the expression

$$\tau_0 = j\, U_\infty. \quad (3)$$

Bearing in mind that during condensation the vapour flows over a moving film surface, we may rewrite the latter expression in the following way:

$$\tau_0 = j(U_\infty - U_s). \quad (4)$$

It is obvious that the condition of validity of equations (3) and (4) is the inequality

$$\frac{2C_Q}{C_f^*} \gg 1. \quad (5)$$

It is worth mentioning that, for the case of a flow over a flat plate with uniform suction of the boundary layer to which condensation at constant heat flux corresponds with a high degree of approximation, condition (3) is a rigorous result of the exact asymptotic solution of the total system of the Navier–Stokes equations [16]. Condition (5) is fulfilled in most cases of vapour flow condensation which are of practical interest. Consequently, in the present work the shearing stress on the external surface of the condensate film is determined from equation (4).

The case of t_w = const.

The case is considered of condensation in longitudinal flow of saturated-vapour with a

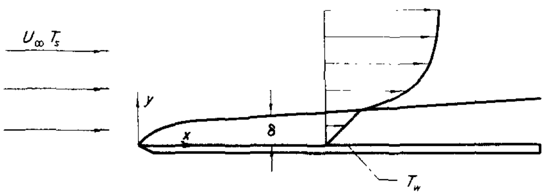

FIG. 1.

constant velocity over a flat plate. It is assumed that the vapour velocity at the infinite distance from the plate is high enough for the gravitational forces to be neglected (for condensation on a horizontal plate or under conditions of weightlessness the analysis will hold also for small vapour velocities). The model of the process and the co-ordinate system are shown in Fig. 1. In the solution, the flow in the liquid film is assumed laminar.

The resistance of the condensate film is taken as the main thermal resistance. In the case considered the assumptions used lead to the constancy of the dimensionless group N.

If inertia forces are neglected, then, with the physical model adopted, the set of equations describing the liquid motion and the heat transfer in a condensate film may be expressed in the following form

$$\left.\begin{aligned} \mu \frac{d^2 U_x}{dy^2} &= 0 \\ \frac{d}{dx}(\bar{U}_x \delta_x \rho) &= \frac{\lambda}{\delta_x} \cdot \frac{\Delta t}{r} \\ \alpha_x &= \frac{\lambda}{\delta_x} \end{aligned}\right\} \quad (6)$$

For the considered case of condensation on an isothermal plate, corresponding boundary conditions will be written as follows

$$\left.\begin{aligned} &U_x = 0; \quad t = t_w = \text{const.} \quad \text{at } y = 0 \\ &\frac{dU_x}{dy} = \frac{j_x}{\mu}(U_\infty - U_s); \quad t = t_s = \text{const.} \\ &\qquad\qquad \text{at } y = \delta_x \\ &\delta_x = 0 \quad \text{at } x = 0 \end{aligned}\right\} \quad (7)$$

The solution of set (6) with the boundary conditions (7) yields the following relations

$$\alpha_x = \tfrac{1}{2}\sqrt{\left(\frac{N}{N+1} \cdot \frac{\rho r U_\infty \lambda}{\Delta t\, x}\right)} \quad (8)$$

$$\bar{\alpha} = \sqrt{\left(\frac{N}{N+1} \cdot \frac{\rho r U_\infty \lambda}{\Delta t\, \mathscr{L}}\right)} \quad (9)$$

The effect of inertia forces

The effect of the inertia forces acting in the film is estimated by means of the following approximate method: the inertia force affecting an elementary volume in the liquid film is determined on the basis of the velocity distribution obtained in the previous solution. The term found in this way, and which describes the inertia force, is introduced into the equation of motion, and the calculation is repeated. The comparison of the present data with the results of the previous analysis yields the following correction factor of the heat-transfer coefficient:

$$C_{in} = \sqrt{\left[0{\cdot}5 + \sqrt{\left(\frac{N^2 + 11N + 5}{20(N+1)}\right)}\right]} \quad (10)$$

As expression (10) shows, in condensation of non-metallic liquids ($N < 0{\cdot}1$), the effect of inertia forces on the heat-transfer rate is negligible.

The convection effect

In film condensation the temperature of the condensed liquid ceases being the same as the saturation temperature, and decreases as the film moves along the cooling surface. This implies that in condensation the cooling surface receives not only latent heat of condensation, but also a certain fraction of the heat content of the liquid already condensed. As is known, this heat quantity is called the supercooling heat. We shall determine the relation between the supercooling heat and the latent heat of condensation. For a linear velocity distribution this relation is

$$\frac{\rho\, C_p\left[\int_0^{\delta_x} \Delta t\, U_x\, \mathrm{d}y - \int_0^{\delta_x} (t - t_w)\, U_x\, \mathrm{d}y\right]}{\rho\, r \int_0^{\delta_x} U_x\, \mathrm{d}y} = \frac{C_p\, \Delta t}{3r} \qquad (11)$$

If it is assumed that the heat quantity $r[1 + \frac{1}{3}(C_p\, \Delta t/r)]$ is released at the external film surface at the moment of the precipitation of the condensate, the supercooling effect is equivalent to that of an increase of the latent heat of condensation. In this situation it is easy to show that the effect of condensate supercooling (due to convection) on heat-transfer rate is characterized by the following correction factor for the heat-transfer coefficient:

$$C_{\text{con}} = \sqrt{\left\{\frac{N + 1 + (N + 1)[(C_p\, \Delta t)/3r]}{N + 1 + [(C_p\, \Delta t)/3r]}\right\}} \qquad (12)$$

As may be seen from expression (12) at $N \ll 1$

$$C_{\text{con}} \cong 1,$$

i.e. in the case of condensation of ordinary liquids, the above correction factor is negligible irrespective of the value of the supercooling heat.

The estimation presented gives rise to certain inaccuracy, since in reality the supercooling heat is given out not at the external film surface at the moment of the precipitation of the condensate but in a subsequent flow of the condensate. The error due to this inaccuracy gives rise to an overestimation or underestimation of the actual heat-transfer coefficient, depending on the direction of the transverse mass flow through the condensate film. The normal component of the velocity vector for the present case of condensation on an isothermal plate is

$$U_n = -\int_0^{y} \frac{\mathrm{d}U_x}{\mathrm{d}x}\, \mathrm{d}y = \tfrac{1}{8} U_\infty \frac{N}{N + 1} \cdot \frac{y^2}{x} \cdot \frac{1}{\sqrt{\left(\frac{N + 1}{N}\right)\frac{\lambda\, \Delta t\, X}{\rho r U_\infty}}} \qquad (13)$$

Expression (13) shows that in the case under consideration, the transverse mass flow is directed from the wall to the film surface. It follows from the above that in the estimate obtained, the effect of convection on heat-transfer rate is somewhat overestimated and its actual effect is even smaller.

The case of q = const.

In some designs of condensers used in engineering, the heat-transfer coefficients on the vapour condensation side exceed those on the opposite side of the heat-transfer surface. The real conditions of the process in these installations correspond, with sufficient approximation, to condensation conditions at a constant heat flux. In view of this, the analysis of the condensation process of the vapour flow at a constant heat flux is also of some interest.

In the present research, the treatment is applied to the case of condensation of non-metallic liquids ($N < 0{\cdot}1$). Proceeding from the results obtained for the case of an isothermal plate, it is assumed that the inertia forces and convection in the film may also be neglected in this case.

The appropriate analysis leads to the following final relations

$$\alpha_x = \sqrt{\left(\frac{\lambda^2 \rho U_\infty}{\mu \mathscr{L}}\right)} \quad (14)$$

$$\bar{\alpha} = \frac{1}{\mathscr{L}} \int_0^{\mathscr{L}} \alpha_x \, dx = 1{\cdot}41 \sqrt{\left(\frac{\lambda^2 \rho U_\infty}{\mu \mathscr{L}}\right)} \quad (15)$$

The results of the present analysis were compared with experimental data of Jakob *et al.* [2] In view of the fact that in these experiments the heat-transfer coefficients on the condensation side of the vapour flow exceed several times those from the other side of the heat-transfer surface, these experiments may be considered as if they were carried out at constant heat flux. At vapour entrance velocities $U > 40$ m/s and temperature drop $\Delta t > 10$ degC, the gravity forces were assumed to be of a minor influence on the process. Bearing in mind that in the experiments just referred to the mean heat-transfer coefficient is defined as $\bar{\alpha}_{exp} = q/\overline{\Delta t}$, we compared the experimental data with the following relation

$$\bar{\alpha}_{exp} = \frac{q}{(1/\mathscr{L}) \int_0^{\mathscr{L}} \Delta t_x \, dx} = 1{\cdot}06 \sqrt{\left(\frac{\lambda^2 \rho U_\infty}{\mu \mathscr{L}}\right)} \quad (16)$$

The comparison is presented in Fig. 2. Experimental data from the experiments carried out in 1933–1934 agreed well with relation (16). The data of the experiments which were carried out a year before fell somewhat higher. In this case the maximum deviation did not exceed 20 per cent. Bearing in mind that the difference between the theory and experiments falls within the scatter of experimental data, the comparison is believed to be quite satisfactory.

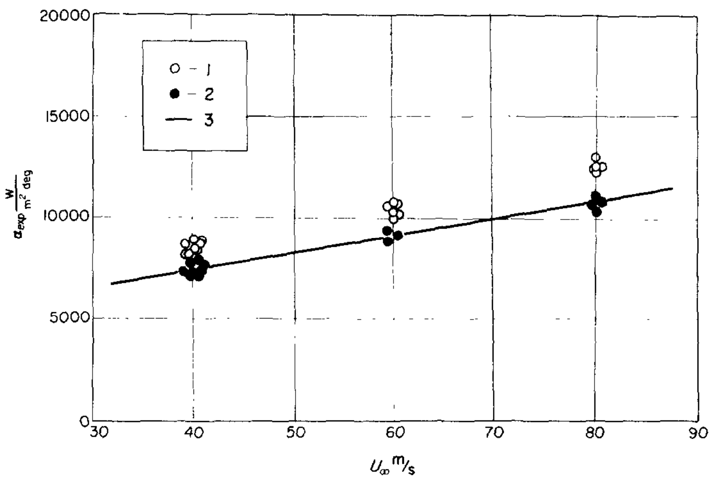

FIG. 2. Comparison of experimental data of Jakob *et al.* [2] with relation (16). (1) experiments of 1932–1933, (2) experiments of 1933–1934; (3) relation (16).

The case of a vertical plate

The case is considered of the condensation in a downward flow of saturated vapour of a nonmetallic liquid over an isothermal surface of a vertical plate. It is easy to observe that in accordance with the results of the previous analysis, the inertia forces and convection in a condensate film may again be neglected. The model of the process and the co-ordinate system are shown in Fig. 3.

The set of equations describing the process

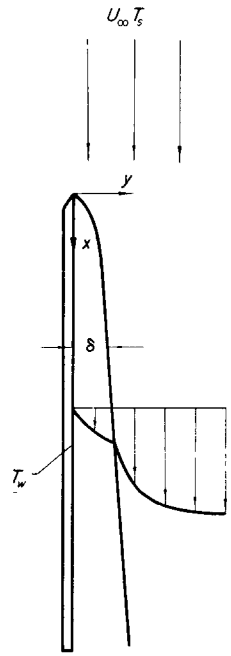

FIG. 3.

considered may be presented in the following form

$$\left.\begin{aligned} \mu \frac{d^2 U_x}{dy^2} + \gamma = 0 \\ \frac{d}{dx}(\bar{U}_x \delta_x \rho) = \frac{\lambda}{\delta_x} \cdot \frac{\Delta t}{r} \\ \alpha_x = \frac{\lambda}{\delta_x} \end{aligned}\right\} \quad (17)$$

The set is solved with the following boundary conditions

$$\left.\begin{aligned} & U_x = 0; \quad t = t_w = \text{const.} \quad \text{at } y = 0 \\ & \frac{d U_x}{dy} = \frac{j_x}{\mu}(U_\infty - U_s); \quad t = t_s = \text{const.} \\ & \qquad \text{at } y = \delta_x \\ & \delta_x = 0 \quad \text{at } x = 0 \end{aligned}\right\} \quad (18)$$

The solution yields

$$\alpha_x = \frac{1}{2}\sqrt{\left(\frac{\lambda^2 \rho U_\infty}{\mu X}\right)}\sqrt{\left[\frac{1 + \sqrt{\left(1 + \frac{16 g X}{U_\infty^2 N}\right)}}{2}\right]} \quad (19)$$

$$\bar{\alpha} = \sqrt{\left(\frac{\lambda^2 \rho U_\infty}{\mu \mathscr{L}}\right)} \frac{\sqrt{2}\left[2 + \sqrt{\left(1 + \frac{16 g \mathscr{L}}{U_\infty^2 N}\right)}\right]}{3\sqrt{\left[1 + \sqrt{\left(1 + \frac{16 g \mathscr{L}}{U_\infty^2 N}\right)}\right]}} \quad (20)$$

3. CONDENSATION ON THE EXTERNAL SURFACE OF A CYLINDER IN A TRANSVERSE FLOW

The laminar film condensation is now considered on the external surface of a cylinder in a transverse vapour flow. The aerodynamics of the process is characterized by a number of special conditions: the flow over the whole cylinder surface is accompanied by a removal of vapour from the boundary layer; vapour flows past a moving condensate film; the boundary-layer separation leads to a sharp change of the condensation rate over a considerable portion of the cylinder.

In the analysis of condensation on the external surface of the cylinder in the present work, the vapour boundary layer is assumed to be laminar up to the separation point. It is also assumed that in comparison with the momentum transferred by the condensing mass, the effect of the pressure gradient along the cylinder periphery may be neglected. Besides, it is assumed that, outside the vapour boundary layer, the velocity field obeys the laws of potential flow around a cylinder. The case is considered of the condensation of the vapour of an ordinary liquid on an isothermal cylinder surface. It is assumed that the effect of the inertia forces may also be neglected in this case. The model of the process and the co-ordinate system are shown in Fig. 4.

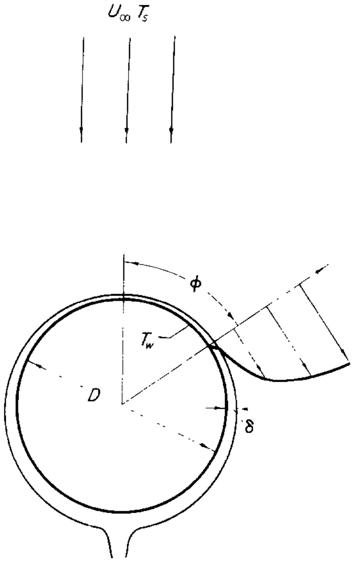

FIG 4.

The case of a cylinder in a flow without separation in the absence of body forces

A flow around a cylinder without separation and with a rather high rate of phase change is also possible for high velocities of the incoming flow. As is known, the removal of vapour due to condensation prevents separation of the boundary layer from the cylinder surface. According to the approximate analysis of Prandtl [17], the suction rate sufficient to prevent separation should satisfy the following inequality:

$$C_Q \sqrt{Re_D} > 4{\cdot}33. \tag{21}$$

With the present simplifying assumptions, the set of equations describing the process under the conditions of weightlessness will be written in the following form

$$\left.\begin{aligned} &\mu \frac{d^2 U_\varphi}{d n^2} = 0 \\ &\frac{2}{D}\cdot\frac{d}{d\varphi}(\bar{U}_\varphi\, \delta_\varphi\, \rho) = \frac{\lambda}{\delta_\varphi}\cdot\frac{\Delta t}{r} \\ &\alpha_\varphi = \frac{\lambda}{\delta_\varphi} \end{aligned}\right\} \tag{22}$$

The set is solved with the following boundary conditions

$$\left.\begin{aligned} &U_\varphi = 0; \quad t = t_w = \text{const.} \\ &\qquad\qquad\qquad \text{at } n = D/2 \\ &\frac{dU_\varphi}{dn} = \frac{j_\varphi}{\mu}(2U_\infty \sin\varphi - U_s); \\ &t = t_s = \text{const.} \quad \text{at } n = \frac{D}{2} + \delta_\varphi \end{aligned}\right\} \tag{23}$$

δ_φ is a finite value at $\varphi = 0$.

The results are expressed in the following form

$$\alpha_\varphi = \frac{\sin\varphi}{\sqrt{(1-\cos\varphi)}}\cdot\sqrt{\left(\frac{\lambda^2\,\rho\,U_\infty}{\mu D}\right)} \tag{24}$$

$$\bar{\alpha} = 0{\cdot}9\sqrt{\left(\frac{\lambda^2\,\rho\,U_\infty}{\mu D}\right)} \tag{25}$$

The process in a gravitational field

Condensation on the external surface of a horizontal cylinder in a transverse downward vapour flow is the most widely used in engineering.

The equation of a laminar flow in a condensate film for the present case will be written in the following form:

$$\mu\frac{d^2 U_\varphi}{dn^2} + \gamma \sin\varphi = 0 \tag{26}$$

The solution of equation (26) with appropriate boundary conditions yields the following expression for the mean condensate velocity in the film

$$\bar{U}_\varphi = U_\infty N \sin\varphi + \frac{\gamma\delta_\varphi^2}{3\mu}\sin\varphi \tag{27}$$

Further analysis of the problem meets mathematical difficulties as the result of which an explicit form of the final solution becomes impossible. However, expression (27) allows us to find, with sufficient approximation, the actual mean value of the heat-transfer coefficient, if the mean coefficients are known (for the same values of all the other parameters) for the case of condensation of vapour at rest in a gravity field and for the case of condensation of moving vapour in the state of weightlessness.

Making use of the known Nusselt formula for condensation of stationary vapour on a horizontal tube and equation (25) which expresses law of condensation of a vapour flowing under the conditions of weightlessness, we obtain the following final relation for the mean heat-transfer coefficient in condensation on a horizontal cylinder in a downward flow of vapour of an ordinary liquid [18]:

$$\bar{\alpha} = 0{\cdot}64\sqrt{\left(\frac{\lambda^2\,\rho\,U_\infty}{\mu D}\right)} \times \sqrt{\left[1 + \sqrt{\left(1 + 1{\cdot}69\,\frac{Dg}{U_\infty^2\,N}\right)}\right]} \quad (28)$$

The analysis of relation (28) shows that the dimensionless group $(Dg/U_\infty^2\,N)$ is a measure of the relative effects of the gravitational and velocity fields on the process in the case considered. It follows from the above that the effect on heat transfer of the motion of condensing vapour must not be described solely in terms of the vapour velocity, and that this effect depends on the value of the temperature difference, besides other parameters. In particular it appears that the heat-transfer intensifying effect of the vapour motion increases as the temperature difference grows. A similar effect of the temperature difference variation on heat transfer has been experimentally found in papers [12–14]. At the same time, it should be noted that according to the theoretical studies [6, 7] based on Nusselt's conceptions, the increase in the temperature difference should lead to the opposite effect, i.e. to a decrease of the intensifying influence of vapour velocity on heat transfer.

In Fig. 5, relation (28) is compared with experimental data of three experimental sets of results [14].† In these experiments the maximum velocity of the incoming vapour was rather small (less than 10 m/s) and it may therefore be assumed that the flow around the tube in these experiments was without separation. Figure 5 shows quite satisfactory agreement between the theory and experimental data.

† According to the note in reference [14], when treating the experimental data of that work, the values of the velocity of the incoming flow were taken to be 3·1 larger than those based on the total cross-section of the experimental channel.

Estimation of the effect of boundary-layer separation

In condensation of vapour flow over the external surface of a horizontal cylinder, boundary-layer separation leads to a sharp decrease of the heat-transfer rate over a portion of the cylinder surface behind the separation point. This is caused by two reasons: first, behind the separation point the vapour begins to flow over the film in the direction opposite to that of the gravitational forces, which results in an increase of the film thickness; secondly, behind the separation point the static pressure becomes rather low as compared with that in the main flow, which fact results in the decrease of the actual temperature difference. Bearing in mind that the lower portion of the cylinder surface, even in a flow without separation contributes little to the total heat transfer (at the surface lying beyond the angle $\varphi = 82°$, 35 per cent of the total quantity of heat is transferred), we may neglect, with sufficient approximation, the heat transfer behind the separation point when the above factors act.

Thus, the mean heat-transfer coefficients for a flow without separation and the flow with a minimum separation angle of $\varphi = 82°$, will differ by 35 per cent. Taking this fact into account, the relation for the calculation of the heat transfer with the minimum separation angle may be presented in the following form

$$\bar{\alpha} = 0{\cdot}42\sqrt{\left(\frac{\lambda^2\,\rho\,U_\infty}{\mu D}\right)} \times \sqrt{\left[1 + \sqrt{\left(1 + 1{\cdot}69\,\frac{Dg}{U_\infty^2\,N}\right)}\right]} \quad (29)$$

Under real conditions separation may begin at any point from 82 to 180° since both the vapour suction and the existence of the moving

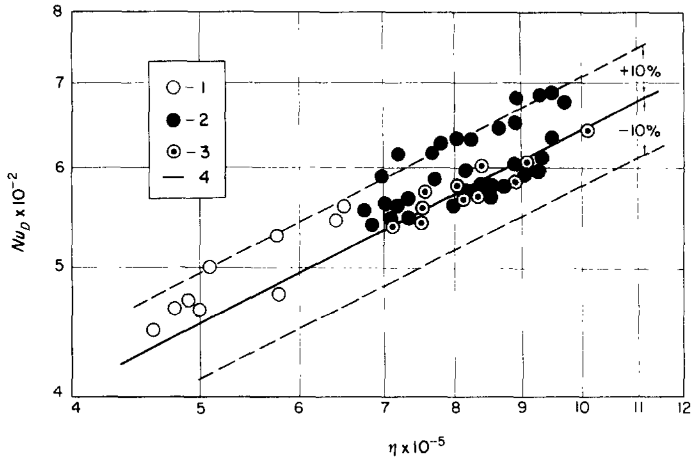

FIG. 5. Comparison of experimental data of Berman and Tumanov [14] with relation (28).
(1) P = 4·7 N/cm², Δt = 7·4 degC; (2) P = 4·7 N/cm², Δt = 2·5 degC; (3) P = 0·31 N/cm², Δt = 1·2 degC; (4) relation (28).

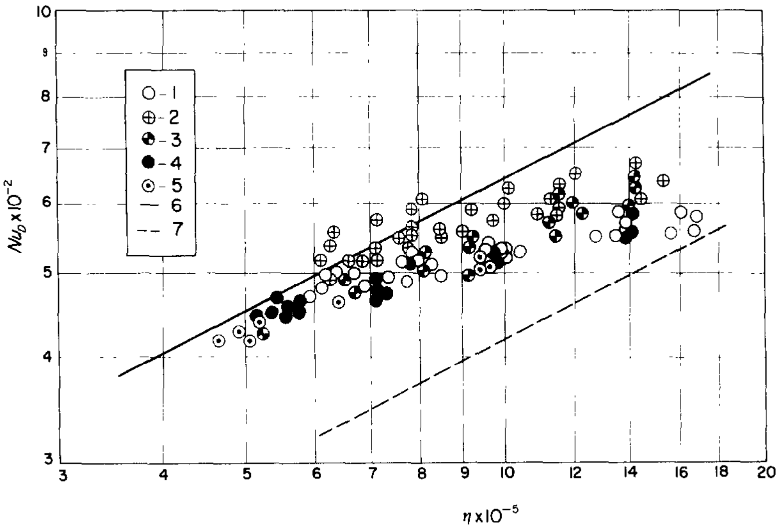

FIG. 6. Comparison of experimental data of Berman and Tumanov [14] with relations (28) and (29).
(1) P = 0·31 N/cm², Δt = 3·1 degC; (2) P = 0·86 N/cm², Δt = 2·2 degC; (3) P = 0·86 N/cm², Δt = 5 degC; (4) P = 0·86 N/cm², Δt = 6·4 degC; (5) P = 0·86 N/cm², Δt = 8·5 degC; (6) relation (28); (7) relation (29).

"vapour–liquid" boundary prevent boundary-layer separation and move the separation point towards the afterbody [19]. It follows from the above that relations (28) and (29) yield the upper and lower limits of the range of the mean heat-transfer coefficient for condensation on a horizontal cylinder under the conditions of vapour flow with a separated boundary layer.

In Fig. 6, relations (28) and (29) are compared with the data of the remaining experiments of reference [14]. In these the maximum vapour velocity reached higher values and the flow around the cylinder was apparently accompanied by a boundary-layer separation. As the comparison of Fig. 6 shows, the experimental points fall between the two theoretical curves. At small values of inflow velocity, with no separation, or when the latter occurs on a small portion of the tube circumference, points fall near the upper curve corresponding to relation (28). As the inflow velocity increases, i.e. as the separation point moves to the angle $\varphi = 82°$, the experimental points approach the lower curve which corresponds to relation (29). Thus, the comparison confirms satisfactorily the results of the present theoretical analysis.

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REFERENCES

1. W. NUSSELT, Die Oberflächenkondensation des Wasserdampfes, *Z. Ver. Dt. Ing.* **60** (27), 541–546; 569–575 (1916).
2. M. JAKOB, S. ERK and H. ECK, Verbesserte Messungen und Berechungen des Warmeüberganges beim kondensieren stromenden Dampfes in einem vertikalen Rohr, *Phys. Z.* **36** (3), 74–84 (1935).
3. S. S. KUTATELADZE, *Condensation and Boiling Heat Transfer (Teploperedacha pri kondensatsii i kipenii).* Mashgiz, Moscow (1952).
4. G. WINCKELSESSER, Die Wärmeabgabe von stromenden Heiss- und Sattdampf. Dechema-Monographien, Bd. 20, Nr. 244, Weinheim (1952).
5. S. N. FUKS, Condensation of a vapour flow over a horizontal tube (Kondensatsiya dvizhushchegosya para na gorizontal'noi trube). *Izv. VTI*, No. 3, 9–13 (1953).
6. S. SUGAWARA, I. MICHIYOSHI and T. MINAMIAMA, The condensation of vapour flowing normal to horizontal pipe, *Proc. 6th Japan Nat. Congr. for Appl. Mech.* III-4 (1956).
7. L. D. BERMAN and YU. A. TUMANOV, The effect of vapour velocity on the mechanism and rate of heat transfer in film condensation on a horizontal tube, *Energomashinostroenie*, No. 5, 24–28 (1964).
8. W. M. ROHSENOW, J. H. WEBBER and A. T. LING, Effect of vapour velocity on laminar and turbulent film condensation, *Trans. Am. Soc. Mech. Engrs* **78**, 1637–1643 (1956).
9. D. CESS, Laminar-film condensation on a flat plate in the absence of a body force, *Z. Angew. Math. Phys.* **11**, 426–433 (1960).
10. J. C. Y. KOH, Film condensation in a forced-convection boundary-layer flow, *Int. J. Heat Mass Transfer* **5**, 941–954 (1962).
11. M. JAKOB, *Heat Transfer*, Vol. 1. John Wiley, New York (1949).
12. L. D. BERMAN and S. N. FUKS, The effect of air admixture on condensation heat transfer in a vapour flow, *Izv. VTI*, No. 11, 5–10 (1952).
13. S. N. FUKS, Condensation heat transfer in a vapour flow in a horizontal tube bundle, *Teploenergetika*, No. 1, 35–39 (1957).
14. L. D. BERMAN and YU. A. TUMANOV, Condensation heat transfer in a vapour flow over a horizontal tube, *Teploenergetika*, No. 10, 77–84 (1962).
15. I. G. SHEKRILADZE, Film condensation in a vapour flow, *Soobshcheniya Akad. Nauk Gruz.SSR*, No. 3, 619–626 (1964).
16. H. SCHLICHTING, *Grenzschicht-Theorie.* Karlsruhe (1951).
17. L. PRANDTL, Mechanics of viscous fluids. In *Aerodynamics* by V. DUREND, Vol. 3. Oborongiz, Moscow (1939).
18. I. G. SHEKRILADZE, Film condensation in a vapour flow. Dissertation. Krzhizhanovsky Power Institute (1965).
19. L. PRANDTL, *Führer durch die Strömungslehre* (1944).

Abstract—Laminar film condensation of a vapour flow along a flat plate and on the external surface of a cylinder in a transverse vapour flow is studied. Momentum transfer is shown to play a dominant part in fluid dynamics of the process itself. The results obtained are compared with the available experimental data.

Résumé—On étude la condensation sous forme de film laminaire d'un écoulement de vapeur le long d'une plaque plane et sur la surface extérieure d'un cylindre dans un écoulement de vapeur transversal. On montre que le transport de quantité de mouvement joue un rôle dominant dans l'aérodynamique du processus lui-même. Les résultats obtenus sont comparés avec les données expérimentales disposibles.

Zusammenfassung—Es wird die laminare Filmkondensation untersucht bei der Dampfströmung entlang einer ebenen Platte und der Querströmung entlang einer ebenen Platte und der Querströmung auf einen Zylinder. Es zeigt sich, dass der Impulsaustausch die dominierende Rolle in der Flüssigkeitsdynamik des Prozesses spielt. Die erhaltenen Ergebnisse werden mit verfügbaren Versuchswerten verglichen.